Amendments to the Specification:

Please amend paragraph 94 to correct the symbols as follows:

[0094] [[≅]] $\underline{\delta_k}(x) = 1$ if x is closest to m_k, otherwise [[≅]] $\underline{\delta_k}(x) = 0$ (resolve ties arbitrarily). The summation of these functions over a data set (see (3) and (4)) residing on the l^{th} unit gives the count, $n_{k,l}$, first moment, [[≅]] $\underline{\Sigma_{k,l}}$, and the second moment, $s_{k,l}$, of the clusters. The vector $\{n_{k,l}$, [[≅]] $\underline{\Sigma_{k,l}}$, $s_{k,l} \mid k=1, \ldots, K\}$, has dimensionality $2 \cdot K + K \cdot \dim$, which is the size of the SS that have to be communicated between the Integrator and each computing unit.

Please amend paragraph 95 to correct the symbols as follows:

[0095] The set of SS presented here is more than sufficient for the simple version of K-Means algorithm. The aggregated quantity, [[≅]] $\underline{\Sigma_k}$ $s_{k,l}$, could be sent instead of the individual $s_{k,l}$. But there are other variations of K-Means performance functions that require individual $s_{k,l}$, for evaluating the performance functions. Besides, the quantities that dominate the communication cost are [[≅]] $\underline{\Sigma_{k,l}}$.

Please amend paragraph 96 to correct the symbols and the equation. In the equation, please note the distinction between the summation symbol and the subscripted variable Σ.

[0096] The l^{th} computing unit collects the SS, $\{n_{k,l}$, [[≅]] $\underline{\Sigma_{k,l}}$, $s_{k,l} \mid k=1, \ldots, K\}$, on the data in its own memory, and then sends them to the Integrator. The Integrator simply adds up the SS from each unit to get the global SS,

$$\cancel{n_k = \sum_{l=1}^{L} n_{k,l}, \quad \sum_k = \sum_{l=1}^{L} \sum_{k,l}, \quad s_k = \sum_{l=1}^{L} s_{k,l}}$$

$$\underline{n_k = \sum_{l=1}^{L} n_{k,l}, \quad \Sigma_k = \sum_{l=1}^{L} \Sigma_{k,l}, \quad s_k = \sum_{l=1}^{L} s_{k,l}.}$$

Please amend paragraph 97 to correct the symbols as follows:

[0097] The leading cost of integration is O(K·dim·L), where L is the number of computing units. The new location of the k^{th} center is given by m_k= [[≅]] $\underline{\Sigma_k}/n_k$ from the global SS (this is the $I($ $)$ function in (2)), which is the only information all the computing units need to start the next iteration. The performance function is calculated by (proof by direct verification),

$$Perf_{KM} = \sum_{l=1}^{L} s_k \, .$$

Please amend paragraph 102 to correct the equation. Note that the numerator in the first summation has changed from "1" to "x".

[0102] (K-Means is similar, except its weights are the nearest-center membership functions, making its centers centroids of the cluster.) Overall then, the recursion equation is given by

$$\cancel{m_k = \sum_{x \in S} \frac{1}{d_{x,k}^3 \left(\sum_{l=1}^{K} \frac{1}{d_{x,l}^2} \right)^2} \Bigg/ \sum_{x \in S} \frac{1}{d_{x,k}^3 \left(\sum_{l=1}^{K} \frac{1}{d_{x,l}^2} \right)^2}}$$

$$\underline{m_k = \sum_{x \in S} \frac{x}{d_{x,k}^3 \left(\sum_{l=1}^{K} \frac{1}{d_{x,l}^2} \right)^2} \Bigg/ \sum_{x \in S} \frac{1}{d_{x,k}^3 \left(\sum_{l=1}^{K} \frac{1}{d_{x,l}^2} \right)^2}} \, . \qquad (9)$$

Please amend paragraph 103 to correct the equation. Note that the index for the summation is "k" rather than "λ", and the denominators in the expression for g_2 and g_3 are cubed "3" rather than taken to the S power "S".

[0103] where $d_{x,k}=\|x-m_k\|$ and s is a constant ≅4. The decomposed functions for calculating SS (see (3) and (4)) are then

$$\cancel{\begin{cases} g_1(x,M)=1\Big/\sum_{\lambda=1}^{K}\frac{1}{d_{x,k}^2} \\ g_2(x,M)=g_1^2(x,M)\cdot\left(\frac{1}{d_{x,1}^S},\frac{1}{d_{x,2}^S},\ldots,\frac{1}{d_{x,K}^S}\right) \\ g_3(x,M)=g_1^2(x,M)\cdot\left(\frac{1}{d_{x,1}^S},\frac{1}{d_{x,2}^S},\ldots,\frac{1}{d_{x,K}^S}\right)x \end{cases}} \quad \underline{\begin{cases} g_1(x,M)=1\Big/\sum_{k=1}^{K}\frac{1}{d_{x,k}^2} \\ g_2(x,M)=g_1^2(x,M)\cdot\left(\frac{1}{d_{x,1}^3},\frac{1}{d_{x,2}^3},\ldots,\frac{1}{d_{x,K}^3}\right) \\ g_3(x,M)=g_1^2(x,M)\cdot\left(\frac{x}{d_{x,1}^3},\frac{x}{d_{x,2}^3},\ldots,\frac{x}{d_{x,K}^3}\right) \end{cases}}$$

Please amend paragraph 108 to correct the equation. Note that the subscript for p is "k" rather than "λ", and the symbol for summation must be carefully distinguished from the subscripted variable Σ.

[0108] In this example, the EM algorithm with linear mixing of K bell-shape (Gaussian) functions is described. Unlike K-Means and K-Harmonic Means in which only the centers are to be estimated, the EM algorithm estimates the centers, the co-variance matrices, Σ_k, and the mixing probabilities, $p(m_k)$. The performance function of the EM algorithm is

$$\cancel{Perf_{EM}(X,M,\Sigma,p)=-\log\left\{\prod_{x\in S}\left[\sum_{k=1}^{K}p_\lambda\frac{1}{\sqrt{(2\pi)^D\det(\sum_k)}}\mathrm{EXP}\left(-(x-m_k)\sum_\lambda^{-1}(x-m_k)^T\right)\right]\right\}}$$

$$\underline{Perf_{EM}(X,M,\Sigma,p)=-\log\left\{\prod_{x\in S}\left[\sum_{k=1}^{K}p_k\cdot\frac{1}{\sqrt{(2\pi)^D\det(\Sigma_k)}}\cdot\mathrm{EXP}\left(-(x-m_k)\Sigma_k^{-1}(x-m_k)^T\right)\right]\right\}} \qquad (13)$$

Please amend paragraph 111 to correct the equation. Note that the symbol for summation must be carefully distinguished from the subscripted variable Σ_k.

[0111] where $p(x|m)$ is the prior probability with Gaussian distribution, and $p(m_k)$ is the mixing probability.

$$\cancel{p(x \mid m_k) = \frac{1}{\sqrt{(2\pi)^D \det(\sum_k)}} \cdot \mathrm{EXP}\left(-(x-m_k)\sum_k^{-1}(x-m_k)^T \right)}$$

$$\underline{p(x \mid m_k) = \frac{1}{\sqrt{(2\pi)^D \det(\Sigma_k)}} \cdot \mathrm{EXP}\left(-(x-m_k)\Sigma_k^{-1}(x-m_k)^T \right)} \qquad (15)$$

Please amend paragraph 112 to correct the equation. Note that the subscript for m is "k" rather than "λ", and the symbol for summation must be carefully distinguished from the subscripted variable Σ.

[0112] M-Step: With the fuzzy membership function from the E-Step, find the new center locations, new co-variance matrices, and new mixing probabilities that maximize the performance function.

$$\cancel{m_k = \frac{\sum_{x\in S} p(m_\lambda|x)\cdot x}{\sum_{x\in S} p(m_k|x)}, \sum_k = \frac{\sum_{x\in S} p(m_k|x)\cdot(x-m_k)^T(x-m_\lambda)}{\sum_{x\in S} p(m_k|x)}, p(m_\lambda) = \frac{1}{|S|}\sum_{x\in S} p(m_\lambda \mid x)}$$

$$\underline{m_k = \frac{\sum_{x\in S} p(m_k|x)\cdot x}{\sum_{x\in S} p(m_k|x)}, \Sigma_k = \frac{\sum_{x\in S} p(m_k|x)\cdot(x-m_k)^T(x-m_k)}{\sum_{x\in S} p(m_k|x)}, p(m_k) = \frac{1}{|S|}\sum_{x\in S} p(m_k \mid x)} . \qquad (16\text{-}18)$$

Please amend paragraph 113 to correct the equation. Note that in the expression for f_1, the subscript for m is "k" rather than "λ".

[0113] The functions for calculating the SS are:

$$\cancel{f_1(x,M,\Sigma,p) = -\log\left[\sum_{l=1}^{K} p(x \mid m_\lambda)p(m_k)\right]}$$

$$\underline{f_1(x,M,\Sigma,p) = -\log\left[\sum_{l=1}^{K} p(x \mid m_k)p(m_k)\right]}$$

$$g_1(x,M,\Sigma,p) = (p(m_1 \mid x), p(m_2 \mid x), \ldots, p(m_K \mid x))$$

$$g_2(x,M,\Sigma,p) = (p(m_1 \mid x)x, p(m_2 \mid x)x, \ldots, p(m_K \mid x)x)$$

$$g_3(x,M,\Sigma,p) = \left(p(m_1 \mid x)x^T x, p(m_2 \mid x)x^T x, \ldots, p(m_K \mid x)x^T x\right)$$